Extended Chandrasekhar Recursions
Ali H. Sayed and Thomas Kailath

Abstract—We extend the discrete-time Chandrasekhar recursions for least-squares estimation in constant parameter state-space models to a class of structured time-variant state-space models, special cases of which often arise in adaptive filtering. It can be shown that the much studied exponentially weighted recursive least-squares filtering problem can be reformulated as an estimation problem for a state-space model having this special time-variant structure. Other applications arise in the multichannel and multidimensional adaptive filtering context.

I. INTRODUCTION

The discrete-time Chandrasekhar recursions for linear least-squares estimation in constant-parameter systems were first presented nearly two decades ago [1]–[4]. The point was that the celebrated Kalman filtering algorithm based on the discrete-time Riccati recursion applied equally to time-invariant, i.e., constant parameter, and time-variant state-space models. This is a strength, but on the other hand one might expect some computational reductions when the model is time-invariant. Replacing the Riccati recursion by the Chandrasekhar recursions does allow such a reduction, from $O(n^2)$ to $O(n^0)$ elementary computations per step, where $n$ is the state dimension. The computational reduction can be very significant in applications where $n$ is quite large (see e.g., [5]–[7]).

There have been some efforts over the years to obtain extensions to time-variant state-space models, and progress in this area has come about through a particular application. In the last few years, there has been a great interest (see e.g., [8]–[12]) in fast versions of recursive least-squares (RLS) algorithms for adaptive filtering and control. These fast RLS algorithms are rather complicated to describe and derive, involving a large number (10–20) of variables and subscripts. In independent work, Houacine et al. [13], [14], and Stock [15] showed that some of these rather complicated fast RLS algorithms could be described and derived much more compactly and simply by recasting the problem in a form to which the Chandrasekhar recursions could be applied. Some manipulation was required to be able to do this because the “natural” model for the problem is not time-invariant; in adaptive filtering the output system matrix is a function of the data, which of course changes with time.

Motivated by this and related problems, we have shown that the Chandrasekhar recursions can be extended to a certain class of time-variant systems in which the time-variation takes place in a certain structured manner. The extended Chandrasekhar recursions are easy to verify, once they have been discovered. This short note is devoted to describing and establishing these extended recursions. Structured time-variations of this sort arise, as mentioned above, in various adaptive filtering problems (and their dual control versions), and may be encountered in other areas as well.

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We first give a brief review of the Riccati-based Kalman filter. Consider a $p \times 1$ process $\{y_i\}$ with an $n$-dimensional state-space model

$$x_{i+1} = F_i x_i + G_i u_i,$$

$$y_i = H_i x_i + v_i \quad \text{for } i \geq 0$$

(1)

where $\{F_i, G_i, H_i\}$ are known matrices with dimensions $n \times n$, $n \times m$, and $p \times n$, respectively. We assume that $x_0, u_i$, and $v_i$ are stochastic variables that satisfy

$$E x_0 = 0, \quad E(x_0 - x_0^*)(x_0 - x_0)^* = \Pi_0, \quad E u_i x_0^* = E v_i x_0^* = 0, \quad E u_i = 0,$$

$$E \begin{bmatrix} u_i^* \\ v_i^* \end{bmatrix} = \begin{bmatrix} Q_i & C_i \\ C_i^* & R_i \end{bmatrix} \delta_{ij}.$$ 

The symbol $\delta_{ij}$ is the Kronecker delta function, $^*$ denotes Hermitian conjugate (complex conjugation for scalars), and $E$ denotes expected value. Let $\tilde{x}_{i-1}$ and $\tilde{y}_{i-1}$ denote the linear least-squares estimates of $x_i$ and $y_i$, given $\{y_0, \ldots, y_{i-1}\}$, respectively. The Kalman filter [16] computes these quantities via the recursions

$$\tilde{x}_{i-1} = H_i \tilde{x}_{i-1},$$

$$\tilde{x}_{i+1} = F_i \tilde{x}_{i+1} + K_i R_i^{-1} e_i,$$

(2)

where $e_i = y_i - H_i \tilde{x}_{i+1}$, $R_{i+1} = E(e_i e_i^*)$, and $K_i = E(\tilde{x}_{i+1} e_i^*)$. Kalman showed that $K_i$ and $R_{i+1}$ can be computed via the equations:

$$K_i = F_i P_{i+1} H_i^* + G_i C_i,$$

and $R_{i+1} = H_i R_{i+1} H_i^* + R_i$, where $P_{i+1}$ is the error covariance in the one-step prediction of $x_{i+1}$, $F_{i+1} \equiv E(x_i - \tilde{x}_{i+1}|x_i - \tilde{x}_{i+1})^*$, and satisfies the Riccati difference recursion:

$$P_{i+1} = \Pi_0 - P_{i+1} F_{i+1}^* R_{i+1}^{-1} F_{i+1} R_{i+1} K_{i+1}^* K_{i+1} R_{i+1}^{-1}.$$ 

(3)

We shall define the square root (factor) of a matrix $A$ as a lower triangular matrix, denoted $A^{1/2}$, such that $A = A^{1/2} A^{1/2}$. We shall also denote $(A^{1/2})^* = A^{1/2}$ and $(A^{1/2})^{-1} = A^{-1/2}$ so that $A^{1/2} = A^{-1/2} A^{1/2}$.

We can check that the number of operations, i.e., multiplications and additions, needed in going from index $i$ to index $(i + 1)$ in the Riccati recursion (3) is $O(n^3)$, and this is true whether or not the state-space model has constant parameters. However, one expects a computationally more efficient procedure in the case of time-invariant (also called constant-parameter) systems $\{F_i, G_i, H_i, Q_i, R_i\}$. Indeed, it has been shown [1]–[4], [17], [18] that in the constant-parameter case the complexity can be reduced to $O(n^2 \alpha)$ per iteration, where the so-called placement rank $\alpha$ is given by

$$\alpha = \text{rank} (FP_{0} F^* + GG^* - K_{p+1} R_{p+1} K_{p+1}^* - \Pi_0) = \text{rank} (P_{i+1} - P_{i+1})$$

This is achieved by using the so-called Chandrasekhar recursions to compute $\{K_i, R_{i+1}\}$ for use in the formulas (2). There are many forms for the Chandrasekhar recursions [1]–[3], but we shall give here the simplest (so-called square-root) version [4].
II. THE SQUARE-ROOT CHANDRASEKHAR FILTER

Let \( \delta P_t = P_{t+1} - P_{t-1} \). It turns out that for constant-parameter systems, the quantity \( \delta P_t \) often has low rank (examples are given later in this section), much less than \( n \), and this fact can be exploited to find a lower complexity algorithm. Observe that \( \delta P_t \) is a Hermitian matrix, so that it has only real eigenvalues. We can factor it (nonuniquely) as

\[
\delta P_t = P_{t+1} - P_{t-1} = L_s S_t L_t^* \tag{4}
\]

where \( S_t \) is an \( \alpha \times \alpha \) signature matrix viz a diagonal matrix with as many \( \pm 1 \)'s on the diagonal as \( \delta P_t \) has positive and negative eigenvalues. In fact, it turns out, as shown ahead, that \( S_t \) is the same for all \( t \), that is \( S_t = S, \forall t \). We now form the prearray

\[
A_t = \begin{bmatrix} R_{t+1/2}^{1/2} & H L_t \\ K_{p_t} & F L_t \end{bmatrix}. \tag{5}
\]

Let \( \Theta_t \) be any \( J = (I \otimes S) \)-unitary matrix \( (\Theta_t \Theta_t^* = J) \) that triangularizes \( A_t \). That is,

\[
A_t \Theta_t = \begin{bmatrix} X & 0 \\ Y & Z \end{bmatrix}
\]

Comparing the entries on both sides of the equality \( A_t J A_t^* = A_t \Theta_t \Theta_t^* A_t^* \) we get

\[
XX^* = R_{t+1/2} = H L_t S_t L_t^* H^*
\]

\[
= (4) R_{t+1} + H (P_{t+1} - P_{t-1}) H^*
\]

\[
= R_{t+1} + R_{t+1} - R_{t+1} = R_{t+1}.
\]

So we can choose \( X = R_{t+1/2} \). Moreover,

\[
YX^* = K_{t+1} + F L_t S_t L_t^* H^*
\]

\[
= K_{t+1} + F (P_{t+1} - P_{t-1}) H^* = K_{t+1}
\]

and hence, we can identify \( Y = K_{p_t+1} \). Finally,

\[
YY^* + ZS_t Z^* = K_{t+1} R_{t+1/2}^{-1} K_{t+1} + F L_t S_t L_t^* F^*
\]

\[
= K_{t+1} R_{t+1}^{-1} K_{t+1} + F (P_{t+1} - P_{t-1}) F^*.
\]

Therefore,

\[
ZS_t Z^* = P_{t+2} - P_{t+1} = L_{t+1} S_t L_t^* \tag{6}
\]

where we have used definition (4). Hence, we can choose \( S_{t+1} = S_t = S \) = signature matrix and identify \( Z = L_{t+1} \). So we are led to the following so-called square-root Chandrasekhar recursions

\[
\begin{bmatrix} R_{t+1/2}^{1/2} & H L_t \\ K_{p_t} & F L_t \end{bmatrix} \Theta_t = \begin{bmatrix} R_{t+1/2}^{1/2} & 0 \\ K_{p_t+1} & L_{t+1} \end{bmatrix} \tag{6}
\]

where \( \Theta_t \) is any \( J = (I \otimes S) \)-unitary matrix that produces the block zero entry on the right-hand side of (6). We can verify that each iteration takes only \( O(n^2 \alpha) \) computations when \( n > p \), as is often the case.

This particular approach to the Chandrasekhar recursions is of course not the way they were originally derived. For more motivation, not necessary here, as to the particular choice of the prearray (5), see [3] and also [19], [22]. Let us consider two special cases [2]:

- \( \mathbf{P}_0 = 0 \): In this case, \( P_{t0} = GG^* \) (assuming \( C_t = 0 \)) and we can choose \( L_0 = GG^* \) and \( S = I \). Moreover, \( \Theta_t \) is any usual unitary matrix.

- \( \mathbf{P}_0 = \mathbf{I} \), that is \( \mathbf{P}_0 \) is the unique nonnegative-definite solution of \( (F \text{ assumed stable}) \mathbf{P} = FF^* + GG^* \). In this case, we get \( P_{t0} - P_{t-1} = -K_{p_t} G L_{t+1} \). So we can choose \( L_0 = K_{p_t} \) and \( S = -I \). Now the matrix \( \Theta_t \) is a \( (I \oplus -I) \)-unitary matrix.

III. STRUCTURED TIME-VARIANT MODELS

The derivation of the Chandrasekhar recursions (6) is based on the fact that \( \delta P_t \) has low rank for constant-parameter systems, as expressed in (4). We now show that these recursions can be extended to a class of time-variant state-space models that exhibit a certain structure in their time-variation.

The computational advantage of the Chandrasekhar recursions stems from the fact that they propagate the low rank factor \( L_t \) instead of \( P_{t+1} \), where \( L_t \) is defined via relation (4). A direct generalization would be to consider differences of the form \( P_{t+1} - \Psi_t P_{t-1} \Psi_t^* \), where the \( \Psi_t \) are convenient time-variant matrices that result in a low rank difference, say of rank \( \alpha \). That is

\[
P_{t+1} - \Psi_t P_{t-1} \Psi_t^* \equiv L_{t+1} S_t L_t^* \tag{7}
\]

for some \( \alpha \times \alpha \) matrix \( L_t \) (we shall also show that for the special time-variant models to be introduced here we shall have \( S_t = S, \forall t \)).

We consider again the state-space model given by (1), and we shall say that it is a structured time-variant model if there exist \( n \times n \) matrices \( \Psi_t \) such that \( F_t, G_t, \) and \( H_t \) vary according to the following rules:

\[
H_t = H_{t+1} \Psi_t, \quad F_{t+1} \Psi_t = \Psi_{t+1} F_t, \quad G_{t+1} = \Psi_{t+1} G_t. \tag{8}
\]

It is clear that constant-parameter systems satisfy (8) with \( \Psi_t = I \). Other special cases of (8) also arise in adaptive filtering as noticed in [20]–[22] and in Section V. We first assume that the covariance matrices \( R_t, Q_t, \) and \( C_t \) are time-invariant whereas \( F_t, H_t, \) and \( G_t \) vary in time according to (8). We shall verify in the next section that these restrictions can be relaxed in order to allow for time-variant \( R_t, Q_t, \) and \( C_t \).

The reason for imposing the conditions specified in (8) will become clear as soon as we give a simple algebraic verification of the proposed recursion (they can also be justified by noting that under these constraints the covariance matrix of the output process viz.

\[
\mathcal{R} = [\text{cov}(\mathbf{y}_t, \mathbf{y}_j)]_{j=1}^t, \quad t \geq 0,
\]

possesses a time-invariant displacement structure as detailed in [19]–[22].

IV. EXTENDED CHANDRASEKHAR RECURSIONS

We derive here the extended Chandrasekhar recursions associated with time-variant models as above, in both the normalized and unnormalized (square-root or array) forms.

A. Square-Root Form

We form the prearray (which should be compared with (5))

\[
A_t = \begin{bmatrix} R_{t+1/2}^{1/2} & H_{t+1} L_t \\ \Psi_{t+1} K_{p_t} & F_{t+1} L_t \end{bmatrix}
\]
and let Θ, be any \( J = (I \otimes S) \) — unitary matrix that triangularizes \( A_i \). That is
\[
A_i \Theta_i = \begin{bmatrix} X & 0 \\ Y & Z \end{bmatrix}.
\]

Comparing the entries on both sides of the equality \( A_i J A_i^* = A_i \Theta_i J \Theta_i^* A_i \), we get (here we use the condition \( H_{i+1} \Psi_i = H_i \))
\[
XX^* = R_{i+1} \Psi_i + H_{i+1} L_i S_i L_i^* H_{i+1}^*
\]
\[
Y \Psi_i^* = \Psi_{i+1}^* K_{i+1} + F_{i+1} L_i S_i L_i^* H_{i+1}^*
\]
\[
Y^* S_i^* = \Psi_i^* + F_{i+1} L_i S_i L_i^* F_{i+1}^*
\]
\[
Z \Psi_i^* = \Psi_{i+1} + F_{i+1} L_i S_i L_i^* F_{i+1}^*
\]
\[
Z^* S_i = \Psi_i^* \Psi_{i+1}^* + F_{i+1} L_i S_i L_i^* F_{i+1}^*
\]

So we can choose \( X = R_{i+1}^{1/2} \). Moreover (we now use the conditions on \( F_i \) and \( G_i \))
\[
YY^* + ZS_i^* Z = \Psi_{i+1} K_{i+1}^* \Psi_i^* + F_{i+1} L_i S_i L_i^* F_{i+1}^*
\]
\[
Y \Psi_i^* = \Psi_{i+1}^* K_{i+1} + F_{i+1} L_i S_i L_i^* H_{i+1}^*
\]
\[
Y^* S_i^* = \Psi_i^* + F_{i+1} L_i S_i L_i^* H_{i+1}^*
\]
\[
Y^* \Psi_i^* = \Psi_{i+1}^* K_{i+1} + F_{i+1} L_i S_i L_i^* H_{i+1}^*
\]
\[
Z \Psi_i^* = \Psi_{i+1} + F_{i+1} L_i S_i L_i^* H_{i+1}^*
\]
\[
Z^* S_i = \Psi_i^* \Psi_{i+1}^* + F_{i+1} L_i S_i L_i^* H_{i+1}^*
\]

and we see that we can choose \( S_{i+1} = S_i \) and idenify \( Z \) as \( L_{i+1} \). Therefore, we are led to the following (square-root) extended Chandrasekhar recursions
\[
\begin{bmatrix} R_{i+1}^{1/2} \\ \Psi_{i+1}^* K_{i+1} \\ F_{i+1} L_i \end{bmatrix} \Theta_i = \begin{bmatrix} R_{i+1}^{1/2} & 0 \\ \Psi_{i+1}^* K_{i+1} & F_{i+1} L_i \end{bmatrix} \Theta_i
\]
\[
(9)
\]

where \( \Theta_i \) is any \( J = (I \otimes S) \) — unitary matrix that produces the block zero entry on the right-hand side of the last expression. These equations should be compared with the Chandrasekhar recursions derived in Section II. The differences are that \( F_{i+1} \), \( H_{i+1} \), and \( \Psi_{i+1} \) appear on the left-hand side of the above expression instead of \( F \), \( H_i \), and \( I \), respectively.

B. Unnormalized Form

It is sometimes convenient to express the extended Chandrasekhar recursions (9) in an unnormalized form. For this, we consider the following alternative factorization (compare with (7))
\[
P_{i+1} \Psi_i^* = \Psi_{i+1} + F_{i+1} L_i S_i L_i^* F_{i+1}^*
\]
\[
\]
\[ u_i = [u_1(i) \ u_2(i) \ \cdots \ u_M(i)], \quad d(i) \text{ and } u_j(i), \ j = 1, \ldots, M, \]
are assumed scalar for simplicity, we are required to determine the linear least-squares estimate of an \( M \times 1 \) column vector of unknown tap weights, \( w = [w_1 \ w_2 \ \cdots \ w_M]^T \), so as to minimize the exponentially weighted error sum
\[
E = (w - \bar{w})^T \Pi_0^{-1} (w - \bar{w}) + \sum_{i=0}^{N-1} \lambda^{N-i} |d(i) - u_i w|^2
\]
where \( \bar{w} = Ew, \ E(w - \bar{w})^T (w - \bar{w}) = \Pi_0 \), and the parameter \( \lambda \) is often called the forgetting factor, since past inputs are exponentially weighted less than the more recent values. In several applications, the input channels exhibit the shift structure: \( u_j(i) = u_{j-1}(i-1) \). That is, if we denote the value of the first channel at time \( i \) by \( u(i) \), then this corresponds to having an input row vector \( u_i \) of the form \( u_i = [u(i) \ u(i-1) \ \cdots \ u(i-M+1)] \). It can be seen (see [20]-[22] for details—see also [13]-[15] for an alternative and related discussion) that this is equivalent to a state-space estimation problem by considering the following \((N+1)\)-dimensional state-space model
\[
\begin{align*}
x_{i+1} &= \lambda^{1/2} x_i, \quad x_0 = [w^T 0]^T
\end{align*}
\]
\[
y(i) = h_i x_i + v(i), \quad E(v(i))^2 = \delta_i
\]
where \( h_i = [h(i) \ u(i-1) \ \cdots \ u(0)]^T (i = 1, \ldots, N) \) is a \( 1 \times (N+1) \) row vector, \( v(i) = d(i)/(\sqrt{\lambda}) \), and \( x_i \) is an \((N+1) \times 1\) state vector with trailing zeros, \((\sqrt{\lambda}) x_i = [w^T 0]^T \). An initial state covariance matrix (with trailing zeros) is assumed viz. \( E(x_0 - \bar{x}_0)(x_0 - \bar{x}_0)^T = \Pi_0 + 0 \), where \( \Pi_0 \) is a \( M \times M \) positive-definite matrix. The corresponding Kalman equations can now be written as
\[
\begin{align*}
\dot{x}_{i+1} &= \lambda^{1/2} x_{i+1} + k_i r_{i-1}^{-1} [y(i) - h_i x_{i+1}]
\end{align*}
\]
\[
r_{i+1} = 1 + h_i P_{i-1} h_i^T, \quad k_i = \lambda^{1/2} P_{i-1} h_i^T h_i^{-1} P_{i-1}
\]
with \( P_{i-1} = \Pi_0 + 0 \). The gain vector \( k_i \) is also has trailing zeros viz. \( k_i = [k_i 0 \cdots 0]^T \). However, though time-variant, the special structure of \( h_i \), viz. \( h_i = h_i + Z \), where \( Z \) is the lower triangular shift matrix, can be further exploited to reduce the operation count to \( O(M) \). Observe that the above relation along with \( F_{i+1} Z = Z F_i \) since \( F_i = \lambda^{1/2} I \) that the state-space model (16) is a special structured time-variant model. The reduction in operation count can now be achieved by using a special case of the extended Chandrasekhar recursions (9) with \( \Psi_i = Z, F_i = \lambda^{1/2} I \). To apply these recursions, we first introduce the (nontrivial) factorization \( L_0 = L_0^T \), where \( L_0 \) and \( S_0 \) are \((N+1) \times \alpha\) matrices, respectively. The factor \( L_0 \) is clearly of the form \( L_0 = [L_1 \ 0]^T \), where \( L_0 \) is \((M+1) \times \alpha\). Let \( \tilde{h}_i \) be the row vector of the first \( M + 1 \) columns of \( h_i \). Writing down the extended Chandrasekhar recursions (9), we obtain
\[
\begin{align*}
\begin{bmatrix}
\bar{x}_{i+1/2} \\
\bar{L}_{i+1}
\end{bmatrix} = \Theta \begin{bmatrix}
\bar{x}_{i+1/2} \\
\bar{L}_{i+1}
\end{bmatrix} +
\begin{bmatrix}
\bar{r}_{i+1/2} \\
\bar{r}_{i+1/2}
\end{bmatrix}.
\end{align*}
\]

VI. CONCLUSION

We have extended the Chandrasekhar recursions to a class of structured time-variant models and we have derived the corresponding square-root (or array) forms in both the normalized and unnormalized forms. An application to the much studied exponentially weighted recursive least-squares filtering problem has been briefly discussed. Further applications to multichannel and multidimensional adaptive filtering, and extensions to alternative windowing schemes will be discussed elsewhere.

REFERENCES

On the Computation of Upper Covariance Bounds for Perturbed Linear Systems

P. Bolzern, P. Colaneri, and G. De Nicolao

Abstract—Motivated by previously published results, the computation of upper covariance bounds for perturbed linear systems is considered. It is shown that, for a wide choice of cost functions, the bound optimization problem is convex with respect to a scalar parameter. The analysis hinges on the properties of a $H_{\infty}$-type Riccati equation.

I. INTRODUCTION

The present note is motivated by the paper [1] where the computation of upper covariance bounds for perturbed linear systems is addressed. Among other things, in [1] it was observed, without any further consideration, that the bound optimization problem might not be convex. The main contribution of the present note is to show that convexity is actually guaranteed for a class of cost functions, including the one considered in [1].

Consider the time-invariant continuous-time linear system

$$\dot{x}(t) = A_0 x(t) + D_0 w(t)$$

where $A_0$ is stable and $w(t)$ is a white noise signal of unit intensity. Then, the asymptotic state covariance $X_0 = X_0 \geq 0$ is the unique solution of the algebraic Lyapunov equation

$$A_0^T X_0 + X_0 A_0 + W_0 = 0$$

where $W_0 = D_0 D_0^T$. As pointed out in [1], perturbations in the system matrix $A_0$ are inevitable in practice, so that the real system matrix is $A_0 + \Delta A$, where $\Delta A$ keeps into account model uncertainties. Correspondingly, as long as $A_0 + \Delta A$ remains stable, the state covariance $X = X'$ of the perturbed system is the unique solution of

$$(A_0 + \Delta A) X + X (A_0 + \Delta A)^T + W_0 = 0.$$ 

In [2] and [1], the problem of obtaining an upper bound for the perturbed state covariance $X$ was dealt with. In particular, it was shown that upper covariance bounds are provided by the solutions of a suitable $H_{\infty}$-type Riccati equation.

Theorem [1]: Let the uncertainty set $\Omega$ be defined as $\Omega \triangleq \{ \Delta A : \Delta A \Delta A^T \leq \bar{A} \}$, where $\bar{A}$ is a given nonnegative matrix. Suppose $A_0$ is stable and $(A_0 + \Delta A, W_0)$ is stabilizable $\forall \Delta A \in \Omega$. If there exist a real $\beta > 0$ and $X \geq 0$ satisfying the Riccati equation

$$A_0 X + X A_0^T + \beta X + \beta \bar{A} + W_0 = 0$$

then $A_0 + \Delta A$ is asymptotically stable and $X \leq X' , \forall \Delta A \in \Omega$.

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